"Pencil and Paper" Assignment 2

Math& 146 - Math 211 (Probability)

Random Variables and Central Limit Theorem

Suppose we have a binomial experiment (two possible outcomes, repeated a number of times, each experiment is identically distributed, and independent of the others), with parameters n (the number of trials), and p (probability of outcome #1). Call the (random) number of outcomes of type #1 N.

Suppose $n = 10, p = 0.6$

1. Write the probability that $N = 8$
2. Write the probability that $N > 8$ (or, equivalently, that $N \geq 9$)
3. You decide to apply a normal approximation to compute the answer to question 2, even if most "rules of thumb", that would support this choice, do not apply. Hence, you will compute the probability that the *average number of outcomes of type #1*, that is $\frac{N}{n}$, will be strictly greater than 8, assuming $\frac{N}{n}$ has a normal distribution with mean p, and variance $\frac{p(1-p)}{n}$.
   a) Justify this statement, under the presumption that we can apply the Central Limit Theorem to this situations
   b) Evaluate the probability that a normal variable with the stated parameters will be strictly greater than 0.8
   c) Evaluate the probability that a normal variable with the stated parameters will be greater or equal to 0.9
   d) For an integer valued variable, like the (precise) binomial we are working with, N, saying that $N > 8$, or $N \geq 9$ is the same thing. For our normal approximation, the previous two choices lead to different numbers. To balance them out, let us compute the probability that a normal variable with the stated parameters will be greater (or equal) to 8.5
4. Compare your results, and draw a conclusion.

Remark: Whatever your final thought may be, note that we are working with a very small n, so that you should not think of drawing any far reaching conclusion from this one case.